



*Algebra, Topology and Analysis:  
 $C^*$  and  $A_\infty$  algebras.*

**Gonio, Batumi, Georgia, August 30 - September 3**



	MONDAY 30
<u>10:00</u> <u>12:00</u>	Registration
12:00	Opening Bakuradze, Purtukhia
<u>12:30</u> <u>13:10</u>	Nadareishvili
<u>13:10</u> <u>13:40</u>	Krutov
13:40	Lunch Break
<u>15:30</u> <u>17:00</u>	Strung
<u>17:00</u> <u>17:20</u>	Coffee Break
<u>17:20</u> <u>18:00</u>	Jacelon

	TUESDAY 31
<u>11:00</u> <u>14:00</u>	Excursion, mountain area
<u>15:00</u> <u>18:00</u>	Conference dinner

WEDNESDAY 1	
10:10 11:40	Strung
11:40 11:50	Q&A session, Coffee
12:00 13:30	Kadeishvili
13:30 13:40	Q&A session
14:00	Lunch Break
15:10 15:50	Jacelon
15:50 16:30	Krutov
16:30 16:50	Coffee Break
16:50 17:30	Grizelj
17:30 17:55	Meerboer

THURSDAY 2	
10:30 12:00	Strung
12:00 12:20	Q&A session, Coffee
12:20 13:40	Kadeishvili
14:00	Lunch Break
15:25 16:05	Ospanov
16:05 16:45	Bokayev
16:45 17:05	Coffee Break
17:05 17:30	Abylayeva
17:30 17:55	Temirkhanova
17:55 18:20	Abekova

FRIDAY 3	
10:10 10:50	Gogatishvili
10:50 11:30	Krutov
11:30 12:10	Taugynbaeva
12:10 12:20	Coffee Break
12:20 12:50	Karlygash
12:50 13:15	Razmadze
14:00	Lunch Break
15:10 16:40	Kadeishvili
16:40 17:00	Q&A session
17:00 17:25	Adamadze
17:30	Closing

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# Presentation

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This summer school is the third in a series of summer schools within the framework of the international doctoral program in mathematics at Tbilisi State University. It will cover wide topics from Algebra, Topology and Analysis, concentrating on  $C^*$  and  $A_\infty$  algebras, it will mainly address graduate students and postdoctoral researchers.

## Organizers

MALKHAZ BAKURADZE  
Tbilisi State University,  
Georgia

AMIRAN GOGITISHVILI  
Czech Academy of  
Sciences, Czech Republic

RALF MEYER  
Georg-August University  
of Göttingen, Germany

## Local Committee

MIKHEIL AMAGLOBELI  
Tbilisi State University,  
Georgia

ANZOR BERIDZE  
Kutaisi International  
University, Georgia

OMAR PURTUKHIA  
Tbilisi State University,  
Georgia

VLADIMIR BALADZE  
Batumi State University,  
Georgia

TENGIZ BOKELAVADZE  
Akaki Tsereteli State  
University, Georgia

RUSLAN SURMANIDZE  
Tbilisi State University,  
Georgia

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# Abstracts of courses

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**Bhishan Jacelon**

*Institute of Mathematics of the Czech Academy of Sciences, Prague, Czechia*

## **Concentration of measure**

Isoperimetric inequalities imply that, probabilistically, Lipschitz functions on high dimensional geometric structures (such as spheres and cubes) are approximately constant. This phenomenon is known as "concentration of measure". In this brief two-lecture introduction, I will describe these geometric situations and discuss examples related to dynamical systems, in particular, groups of measure preserving automorphisms of Lebesgue space, and Anosov diffeomorphisms like Arnold's cat map.

# Tornike Kadeishvili

*A. Razmadze Mathematical Institute of Tbilisi State University*

## $A_\infty$ algebra Structure in Cohomology and its Applications

The main method of algebraic topology is to assign to a topological space certain algebraic object (model) and to study this relatively simple algebraic object instead of complex geometric one. Examples of such models are chain and cochain complexes, homology and homotopy groups, cohomology algebra, etc. The main problem here is to find models that classify spaces up to some equivalence relation, such as homeomorphism, homotopy equivalence, rational homotopy equivalence (an equivalence relation generate by maps that induce isomorphisms of rational homology), etc.

Usually such models are not complete: the equivalence of models does not guarantee the equivalence of spaces. They can just distinguish spaces. The models which carry richer algebraic structure contain more information about the space. For example the model “cohomology algebra” allows to distinguish spaces, which can not be distinguished by the model “cohomology groups”.

One can not expect the existence of more or less simply complete algebraic models in general case but for the rational homotopy equivalence there are various complete homotopy invariants due to Quillen and Sullivan. The key point here is the existence in the rational case of commutative cochains. Two 1-connected spaces are rationally homotopy equivalent if and only if their commutative cochain algebras are weak equivalent .

But outside of rational case the situation is much more complicated. The ordinary (noncommutative) cochain complex is too poor to determine homotopy type. The structure of differential graded algebra must be enriched with new cochain operations, such as Steenrod operations, which measure the deviation from commutativity. But this structure also is not enough. The further enrichment requires huge structure of cochain operation. The operadic technics is appropriate tool to handle such huge structures. The final result in this direction is the result of Mandell stating that for some class of topological spaces cochain complex equipped by a structure of algebra over so called  $E_\infty$ -operad determines homotopy type

## **Andrey Krutov**

*Institute of Mathematics of the Czech Academy of Sciences, Prague, Czechia*

### **Introduction to dynamical systems**

During the course we will discuss the classical dynamical systems. In particular, for Hamiltonian dynamical systems will provide, the classical integrability criteria and investigate some topological properties of their solutions.

## **Karen Strung**

*Institute of Mathematics of the Czech Academy of Sciences, Prague, Czechia*

### **Smale spaces and their C\*-algebras**

Smale spaces are a class of hyperbolic dynamical system defined by Ruelle to model the behaviour of the restriction of a so-called "Axiom A diffeomorphism" to its basic sets. Examples include shifts of finite type, certain tiling spaces, and Anosov diffeomorphisms. From a Smale space we can construct several C\*-algebras which capture different aspects of the dynamics. This talk will explore the interactions between Smale spaces and C\*-algebras: after introducing Smale spaces, we will see various examples, and then see how to use the dynamics to construct C\*-algebras. We will see that properties like simplicity of the C\*-algebra coincides to the Smale space being mixing. Time permitting, I will also present further properties of these C\*-algebras.



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# Titles and Abstracts of talks

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**Azhar Abekova**

*L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan*

**On cones of nonincreasing functions generated by generalized fractional maximal functions**

**Davit Adamadze**

*Tbilisi State University, Georgia*

**Some examples of singular extensions of order continuous functionals**

In this talk we investigate a boundedness of the Hardy-Littlewood maximal operator in the variable Lebesgue spaces when  $p_+ = \infty$ . Result obtained here is a generalization of Lerner's theorem.

**Nurzhan Bokaev**

*L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan*

**On boundedness and compactness of Commutators for singular integrals on Global Morrey-type spaces**

**Karmen Grizelj**

*University of Zagreb, Zagreb, Croatia*

**Harish-Chandra map and primitive invariants**

## **Stein Meereboer**

*Utrecht University, Utrecht, Netherlands*

### **Crossed products of Banach algebras**

## **George Nadareishvili**

*Tbilisi State University, Georgia*

### **Approximations of Kasparov categories of noncommutative spaces**

This talk will be a very short overview of noncommutative topology. The field being vast, we will concentrate on the theory of abelian approximations of Kasparov categories of noncommutative spaces using relative homological algebra.

If time permits, we will also mention recent results on classification of relevant subcategories and universal coefficient spectral sequences (ongoing joint work with Ralf Meyer, Georg-August University of Göttingen).

## **Korgan Ospanov**

*L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan*

### **Maximal Regularity of a System of Second-Order Difference Equations**

## **Konstantine Razmadze**

*Tbilisi State University, Georgia*

### **Analysis of the multi-modal logics for modal maps**

We axiomatize various bi-modal logics arising from maps between Kripke frames or maps between topological spaces. One modality is interpreted using the relational or topological structure, while the other is interpreted using the mapping involved. For some of the axiomatized logics we also prove the finite model property, which implies their decidability.

**Bakjan Tuerdebieke**

*L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan*

**On products of noncommutative symmetric quasi Banach spaces and applications**